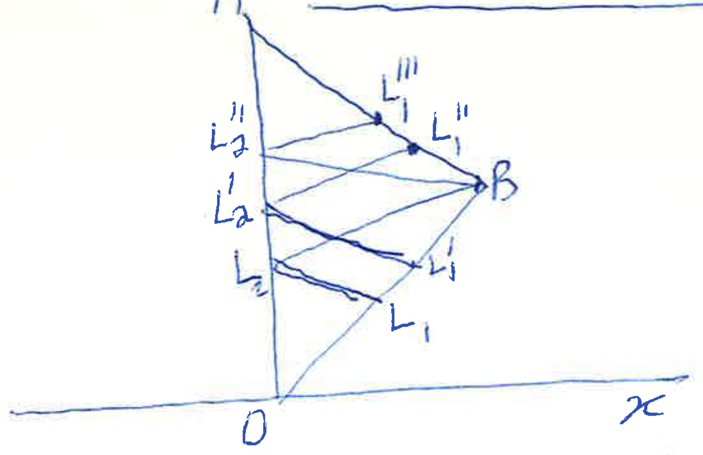
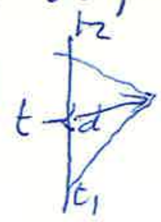
THE ETERNITY MACHINE ①



I moves with velocity V along OB – returns along BA



$$t=\tfrac12(t_2+t_1)$$
$$d=\tfrac12 c(t_2-t_1)$$

Eq. OB is $t=\frac{x}{v}$ $\quad OL_2'=T,\quad OA=2T$

$B=(vT,T)$, $O=(0,0)$, $A=(0,2T)$

$$OB=\sqrt{T^2-\frac{v^2T^2}{c^2}}=T\sqrt{1-v^2/c^2}$$

$$OB+BA=2T\sqrt{1-v^2/c^2}$$

Eqn of L_2B is $(t-T)=\frac1c(x-vT)$

So $L_2=(0,T(1-v/c))$

Eq. of L_2L_1 is $(t-T(1-v/c))=-\frac1c(x)$

∴ L_1 = soln of $\left.\begin{array}{l} t=x/v \\ t=T(1-v/c)-\frac{x}{c}\end{array}\right\}\begin{array}{l} x=T\left(\frac{c-V}{c+V}\right)v \\ t=T\left(\frac{c-V}{c+V}\right)\end{array}$

$$t(L_2)=\tfrac12\left(T+T\frac{c-V}{c+V}\right)=T\cdot\frac{c}{c+V}$$

$$\sigma\, t(L_2)=\sqrt{1-v^2/c^2}\cdot T\cdot\frac{c}{c+V}=\sqrt{\frac{c-V}{c+V}}\cdot T=\frac{1}{\sqrt{1-v^2/c^2}}T(1-v/c)$$

and slope of $L_2, t(L_2)=\dfrac{\left(T\cdot\frac{c}{c+V}-T(1-V/c)\right)}{\frac12\left(vT+T\left(\frac{c-V}{c+V}\right)v\right)}=v/c^2$ ✓

and $d(L_2)=\frac12 c(OB-OL_1)=\frac12 c\,T\left(\sqrt{1-v^2/c^2}-\sqrt{1-v^2/c^2}\left(\frac{c-V}{c+V}\right)\right)$

$$=\tfrac{2}{2}cT\sqrt{1-v^2/c^2}\cdot\frac{V}{c+V}$$

$$=\cancel{T(1-v/c)\cdot Vc\sqrt{1-v^2/c^2}\cdot\frac{1}{(1-v/c)^2}}$$

$$=Tv\sqrt{1-v^2/c^2}\cdot\frac{c}{c+V}$$

Now write: $L_2 L_2' = \alpha$.

so $O L_2' = T(1-v/c) + \alpha$.

α ranges from 0 to $2vT/c$.

Eq. of $L_1' L_2'$ is $\left(t - T(1-v/c) - \alpha\right) = -\frac{1}{c}(x)$

Eq. of $L_2' L_1''$ is $\left(t - T(1-v/c) - \alpha\right) = \frac{1}{c} x$

$L_1' L_2'$ intersects OB

where $\left.\begin{aligned}\left(t - T(1-v/c) - \alpha\right) &= -1/c \cdot x \\ t &= x/v\end{aligned}\right\}$

i.e. $t - T(1-v/c) - \alpha = -\frac{1}{c} vt$

or $t = \dfrac{T(1-v/c) + \alpha}{1+v/c}$

and $L_2' L_1''$ intersects AB where

$\left.\begin{aligned}\left(t - T(1-v/c) - \alpha\right) &= \frac{1}{c} x \\ t - 2T &= -1/v\, x\end{aligned}\right\}$

~~or $t = T(1-v/c) + \alpha \neq (t-2T)$~~

~~or $t = T(1-v/c) + \alpha + 2T$~~

or $t =$

$t = T(1-v/c) + \alpha + \frac{1}{c}\left(-v(t-2T)\right)$

or $t\left(1+\frac{v}{c}\right) = T(1+v/c) + \alpha$

or $t = \dfrac{T(1+v/c) + \alpha}{1+v/c}$

$$\therefore \quad \frac{t_1+t_2}{2} = \frac{T+d}{1+v/c}$$

and $t(d) = \sqrt{1-v^2/c^2} \cdot \dfrac{T+d}{1+v/c}$

when $d=0$ $\quad t(d) = \sqrt{\dfrac{c-v}{c+v}} \cdot T$

$$= \frac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c)$$

slope of $t(d)$ line is $\sqrt{\dfrac{c-v}{c+v}} = \sqrt{\dfrac{1-v/c}{1+v/c}}$

So time lapse for (1)

is up to $T(1-v/c)$: $\dfrac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c)$

from $T(1-v/c)$ to $T(1+v/c)$: $2Tv/c\sqrt{\dfrac{c-v}{c+v}}$

from $T(1+v/c)$ to $2T$: $\dfrac{1}{\sqrt{1-v^2/c^2}}\, T(1-v/c)$

$$\text{Sum} = 2T\sqrt{\frac{c-v}{c+v}}\,(1+v/c)$$

$$= \frac{2T}{c}\sqrt{c^2-v^2} = 2T\sqrt{1-v^2/c^2}.$$

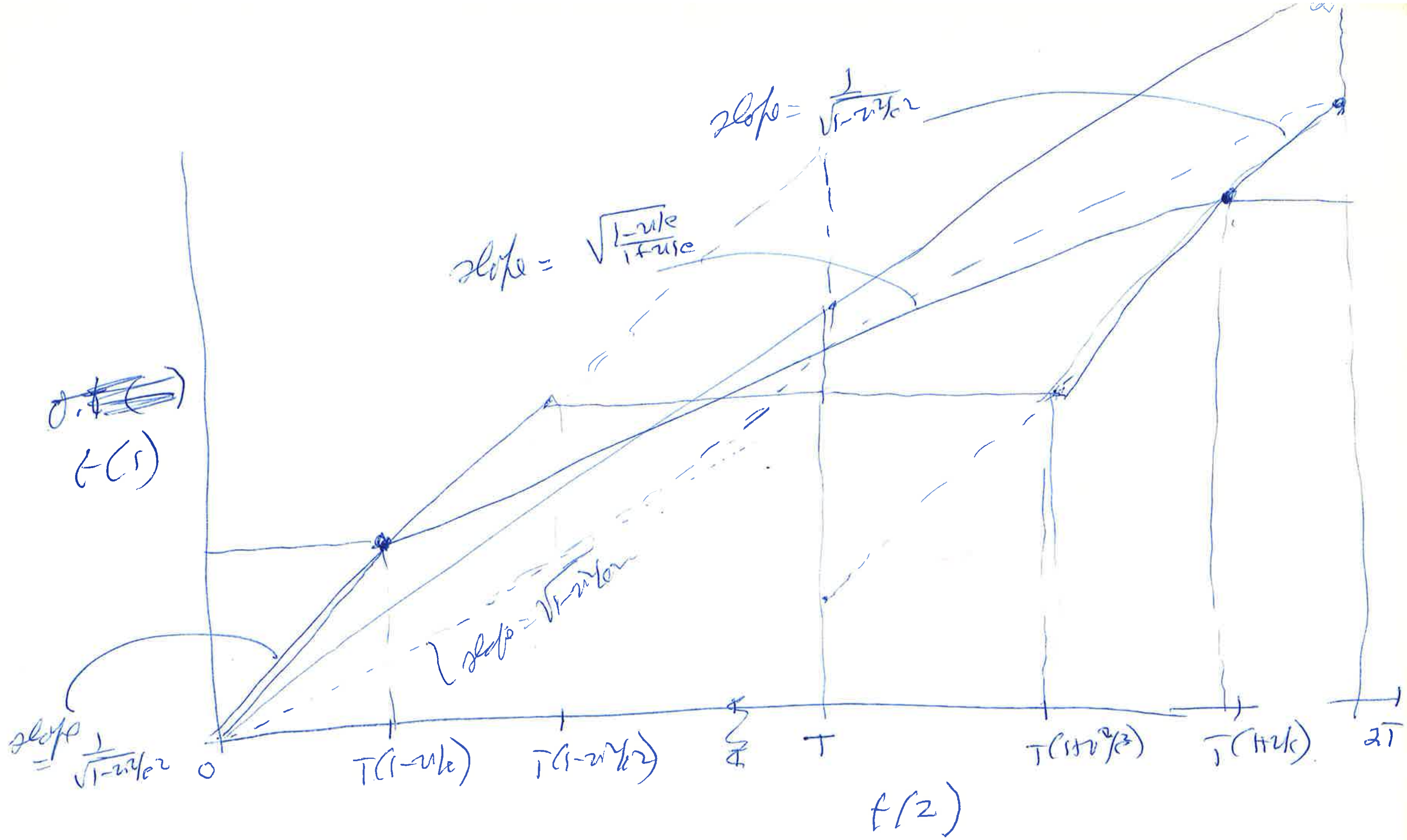
time for (1) up to T is $\dfrac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c) + T\cdot\sqrt{1-v^2/c^2}.$

$= 2T\sqrt{1-v^2/c^2}$

obtained by putting $d = v/c$

$\Rightarrow \quad t(T) = \sqrt{1-v^2/c^2} \cdot T$

④


slope $= \frac{1}{\sqrt{1-v^2/c^2}}$
slope $= \sqrt{\frac{1-v/c}{1+v/c}}$
$t(1)$
slope $= \sqrt{1-v^2/c^2}$
slope $= \frac{1}{\sqrt{1-v^2/c^2}}$
0
$T(1-v/c)$
$T(1-v^2/c^2)$
T
$T(1+v^2/c^2)$
$T(1+v/c)$
$2T$
$t(2)$

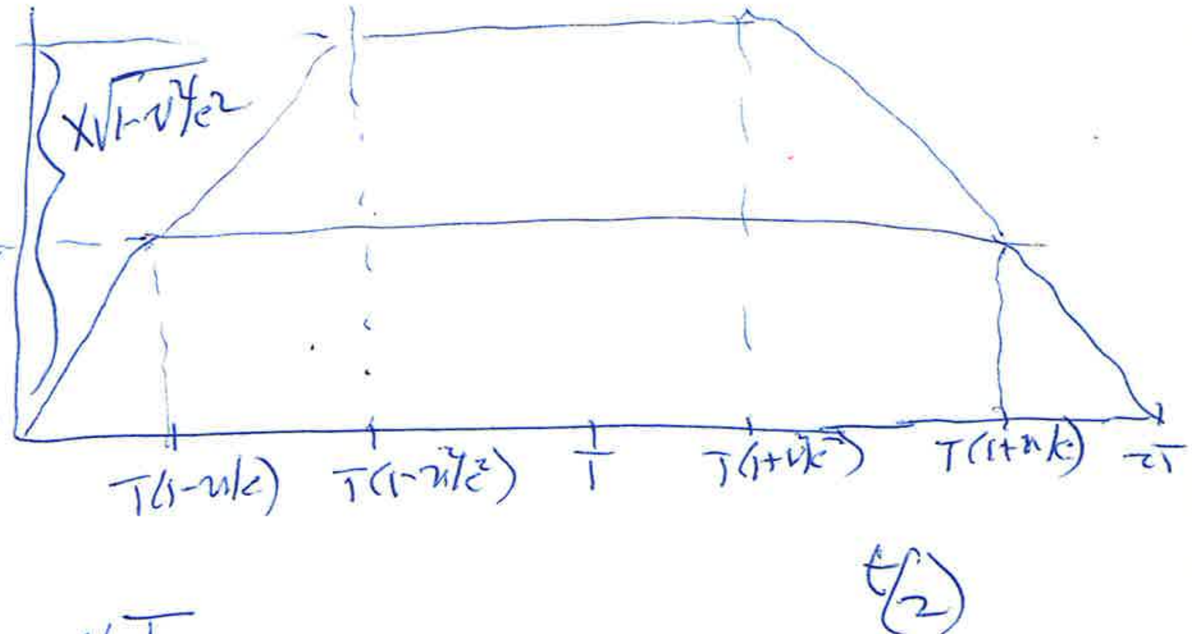
Notice that

$$\frac{1}{2}c\left(t_2 - t_1\right) = \frac{\cancel{2}T v\cancel{/c}}{1+v/c}$$

So $d(\alpha) = \sqrt{1-v^2/c^2}\cdot \cancel{\frac{2T v/c}{1+v/c}}\ \frac{Tv}{1+v/c}$

(v independent of α.

$$= T\cdot v\sqrt{1-v^2/c^2}\cdot\frac{c}{c+v}$$

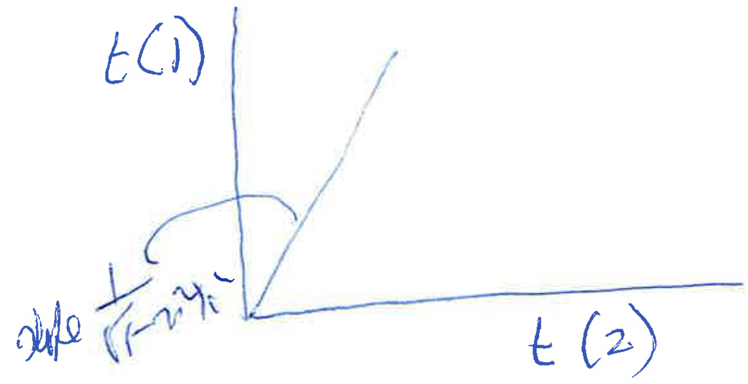


$\cancel{X}\sqrt{1-v^2/c^2}\cdot\frac{c}{c+v}$
$= X\sqrt{\frac{c-v}{c+v}}$
$= X\sqrt{\frac{1-v/c}{1+v/c}}$

where $X = vT$

slope of $(d, t/2)$ curve is $\frac{X}{T}\frac{\sqrt{c^2-v^2}}{c+v}\cdot\frac{1}{1-v/c} = v\cdot\frac{c}{\sqrt{c^2-v^2}}$
$= \frac{v}{\sqrt{1-v^2/c^2}}$

Compute clock → E-parameter or m-value



(1) change t(1) from proper time to 2's coord. time.

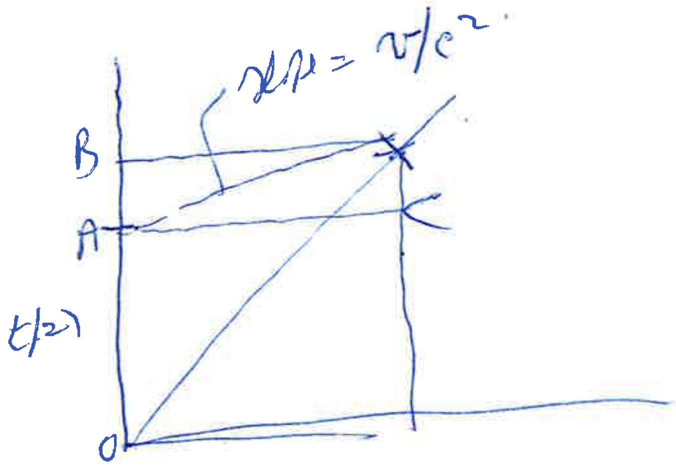
then slope $\rightarrow \frac{1}{\sqrt{1-v^2/c^2}} \times$ old slope

$$= \frac{1}{1-v^2/c^2}.$$

s' = slope with proper time for (2)

$$= \sqrt{1-v^2/c^2} \cdot s$$

or $s = \frac{s'}{\sqrt{1-v^2/c^2}}$



slope of t(1) v t(2) curve in 2's coord. time

$$= \frac{OB}{OA} = \frac{OA + AB}{OA} = 1 + \frac{AB}{OA}$$

slope m of line of simultaneity $= \frac{AB}{AC} = \frac{OB - OA}{OB \times v}$

$$= \frac{1}{v}\left(1 - \frac{OA}{OB}\right)$$

$$= \frac{1}{v}\left(1 - (1 - v^2/c^2)\right)$$

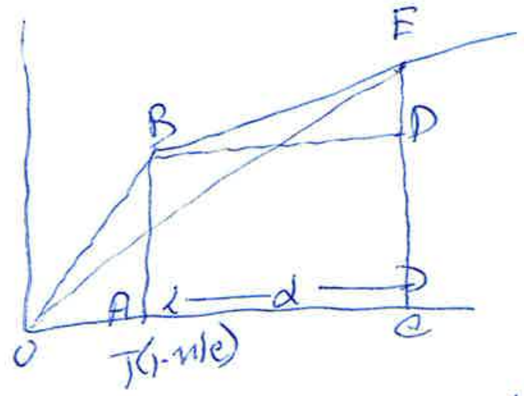
$$= v/c^2$$

So $m = \frac{1}{v}\left(1 - \frac{1}{s}\right)$

$$= \frac{1}{v}\left(1 - \frac{\sqrt{1-v^2/c^2}}{s'}\right)$$

along composite dash portion of $f(s)-f(1)$ curve (7)

$$S' = \frac{Ee}{Oe}$$



$$= \frac{ED + De}{OC} = \frac{\frac{1}{\sqrt{1-v^2/c^2}} \cdot T(1-v/c) + \sqrt{\frac{1-v/c}{1+v/c}}\left(t - T(1-v/c)\right)}{t}$$

$$= \sqrt{\frac{1-v/c}{1+v/c}} + \frac{T}{t}\left(\sqrt{\frac{1-v/c}{1+v/c}} - \sqrt{\frac{1-v/c}{1+v/c}}\,(1-v/c)\right)$$

$$= \sqrt{\frac{1-v/c}{1+v/c}} + \frac{T}{t}\cdot\frac{v}{c}\sqrt{\frac{1-v/c}{1+v/c}}$$

$$= \sqrt{\frac{1-v/c}{1+v/c}}\left[1 + \frac{T}{t}\cdot\frac{v}{c}\right]$$

and $m = \frac{1}{v}\left(1 - \sqrt{1-v^2/c^2}\,\sqrt{\frac{1+v/c}{1-v/c}}\left(1+\frac{T}{t}v/c\right)^{-1}\right)$

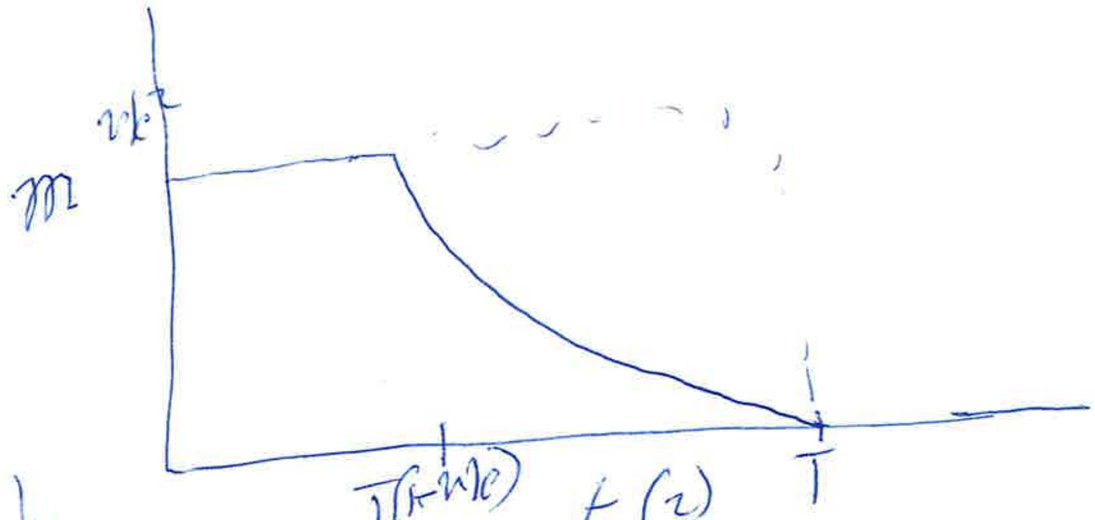
$$= \frac{1}{v}\left(1 - \frac{1+v/c}{1+\frac{T}{t}\cdot v/c}\right) \quad //$$

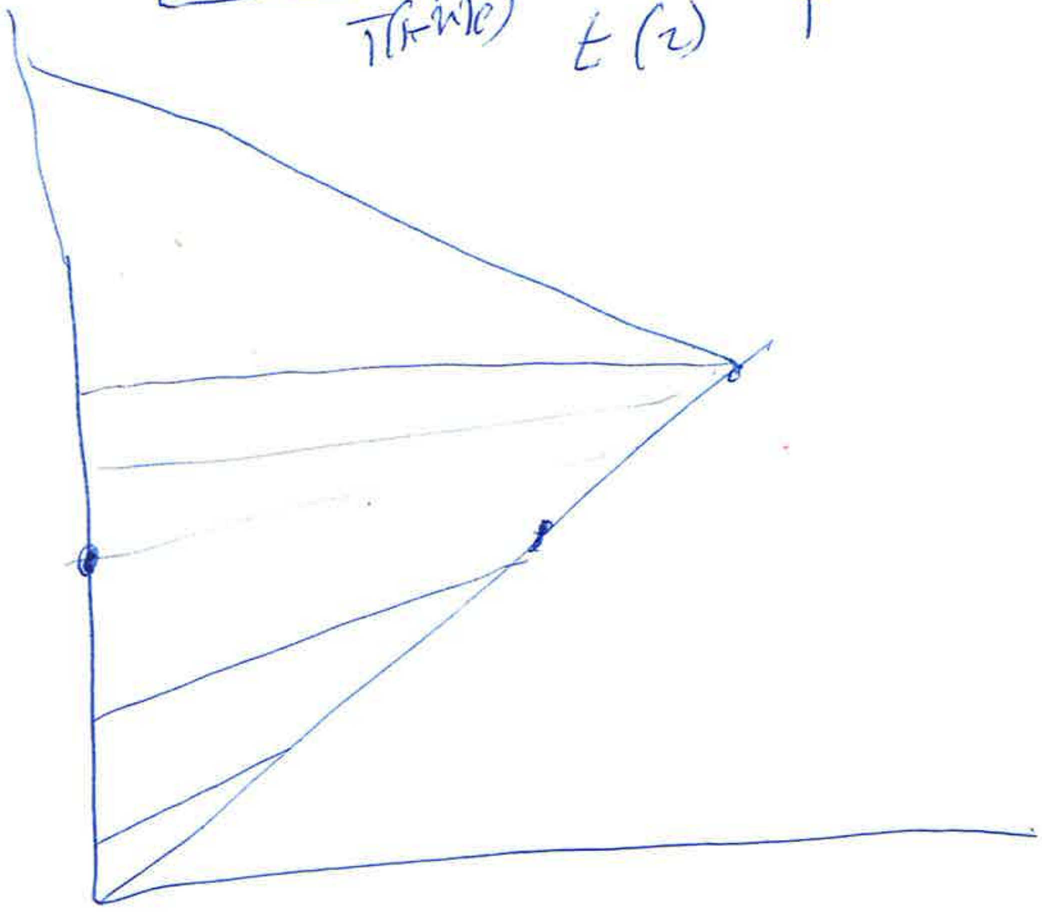
when $t = T$, $m = 0$

when $t = T(1-v/c)$, $m = \frac{1}{v}\left[1 - \frac{1+v/c}{1+\frac{v/c}{1-v/c}}\right]$

$$= \frac{1}{v}\left[1 - (1-v^2/c^2)\right] = v/c^2.$$

Now, the connection between m and ε is





P.T.P. relevant $\dfrac{d'(\rho)}{\tau'(\rho)}$ of (2) at seen point

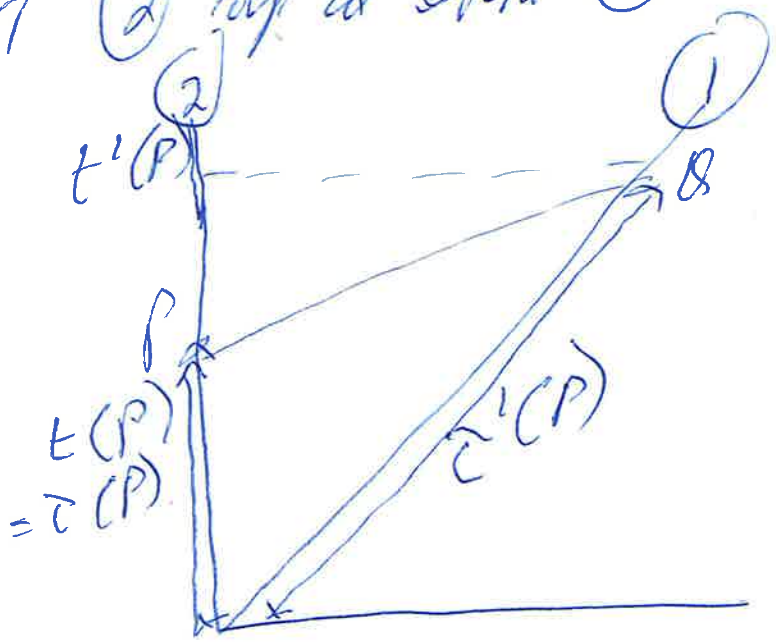
$\to 0$ since $d'(\rho)$ remains constant for $t(\rho) > T(1-\nu/\rho)$, $< T(1+\nu/\rho)$

Improved notation

$t'(P)$ is time on world line of 1 (point Q) which is judged simultaneous with event P on worldline of 2, using 2's time coordinate

$\tau'(P)$ is proper time up to $t'(P)$ as measured by 1

$t(P) = \tau(P) =$ time (& proper time) measured by 2 up to event 2



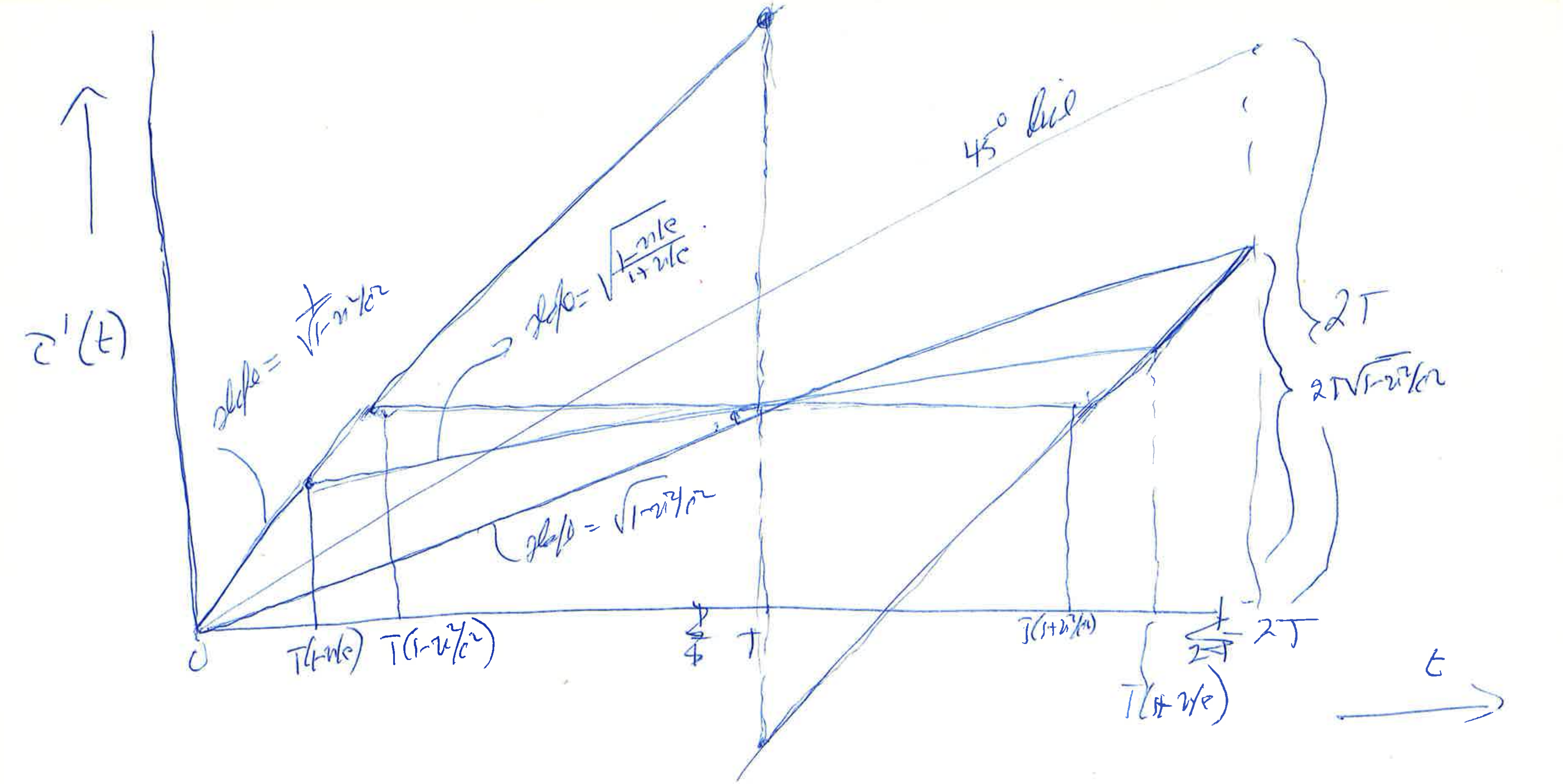
Then our basic diagram plots $\tau'(P)$ versus $\tau(P)$ as P moves along world line of 2

$d'(P)$ is distance of 2 from 1 as seen from 1 at its proper time $\tau'(P)$

$s =$ slope of $t'(P) - t(P)$ curve

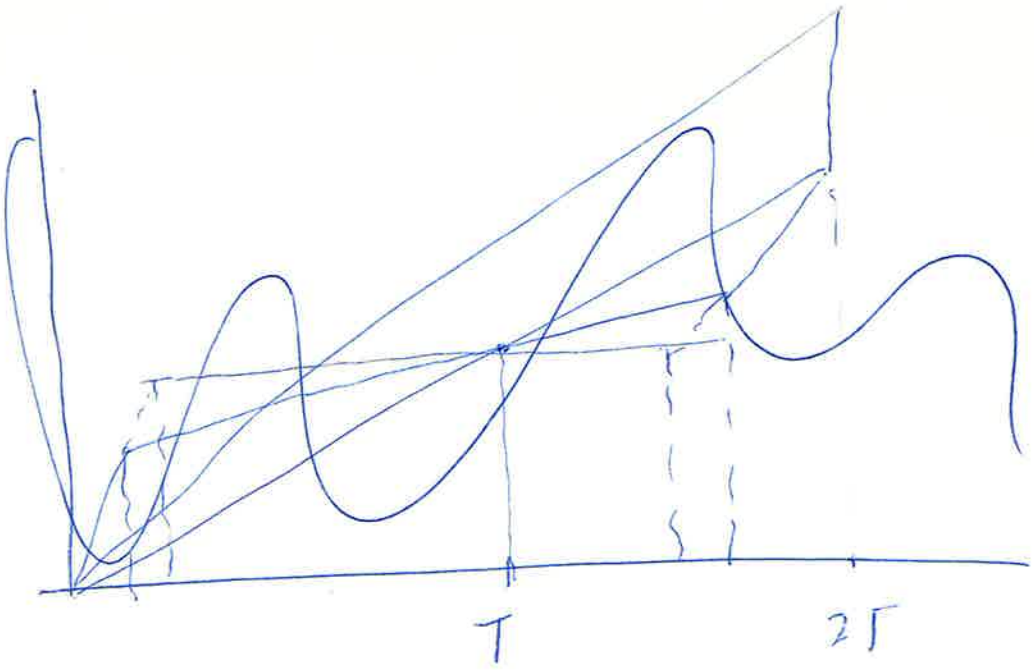
$s' =$ slope of $\tau'(P) - \tau(P)$ curve.

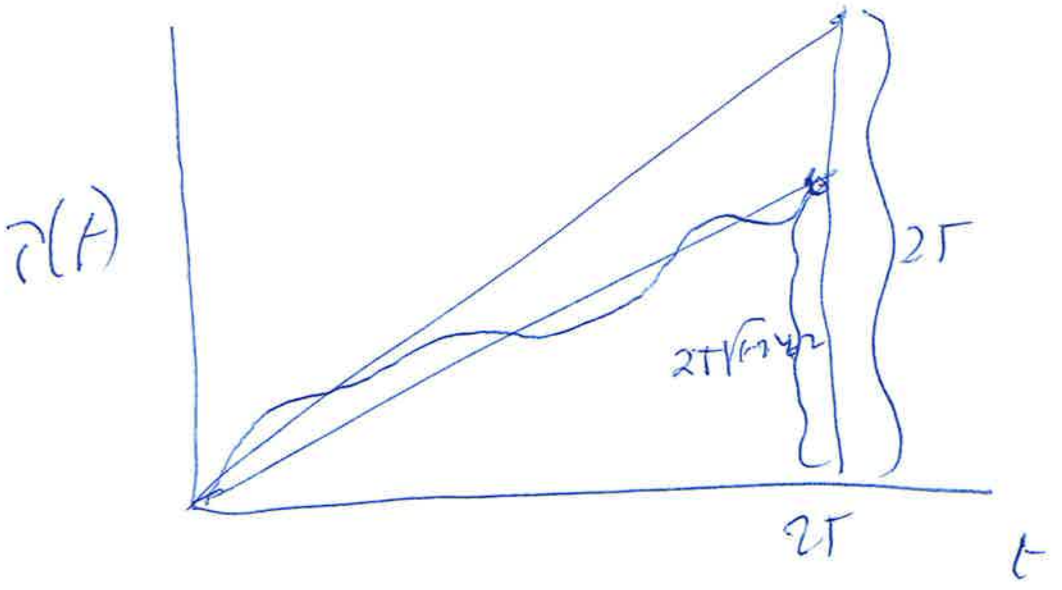
$m =$ slope of PQ in 2's reference frame.



$\tau'(t)$
slope $= \frac{1}{\sqrt{1-v^2/c^2}}$
slope $= \sqrt{\frac{1-v/c}{1+v/c}}$
slope $= \sqrt{1-v^2/c^2}$
45° line
$2T$
$2T\sqrt{1-v^2/c^2}$
0
$T(1-v/c)$
$T(1-v^2/c^2)$
T
$T(1+v^2/c^2)$
$T(1+v/c)$
$2T$
t

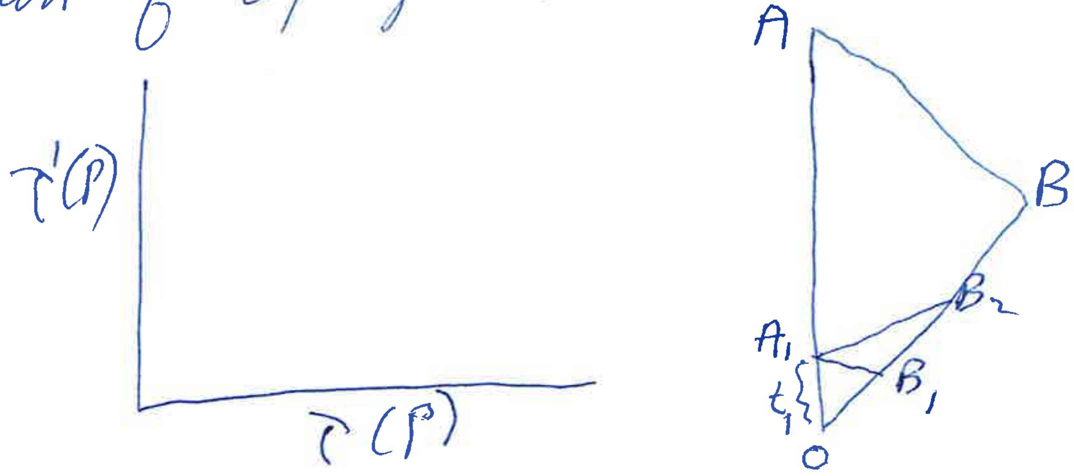
(10)





arbitrary synchronization case:
cp. special cases in previous diagram

we want now to determine upper and lower limits $\tau'_u(P)$ and $\tau'_e(P)$ of proper time for ① that is ~~firm~~ judged simultaneous with P on world line of ① by criterion of topological simultaneity



at point A, so $0A = t_1$, Eq. of A_1B_1 is

$t - t_1 = -\frac{1}{c}x$, intersects $0B$ with Eq. $t = x/v$

at ~~time~~ x-value $x/v - t_1 = -1/c\, x$.

or $x\left(\frac{1}{v}+\frac{1}{c}\right) = t_1$, so t-value is

$\frac{1}{v}\left(\frac{t_1}{\frac{1}{v}+1/c}\right)$ and $\tau'_e(t_1) = \sqrt{1-v^2/c^2}\cdot c\,\frac{t_1}{c+V}$

$\boxed{\text{slope} = \sqrt{\frac{1-\beta}{1+\beta}}}$

Eq. of A_1B_2 is $t - t_1 = \frac{1}{c}x$, intersects $0B$ with Eq. $t = x/v$

at x-value, $x\left(\frac{1}{v}-\frac{1}{c}\right) = t_1$ so t-value

is $\frac{1}{v}\left(\frac{t_1}{\frac{1}{v}-\frac{1}{c}}\right)$ and $\tau'_u(t_1) = \sqrt{1-v^2/c^2}\cdot c\,\frac{t_1}{c-V}$

$\boxed{\text{slope} = \sqrt{\frac{1+\beta}{1-\beta}}}$

these formulas apply until

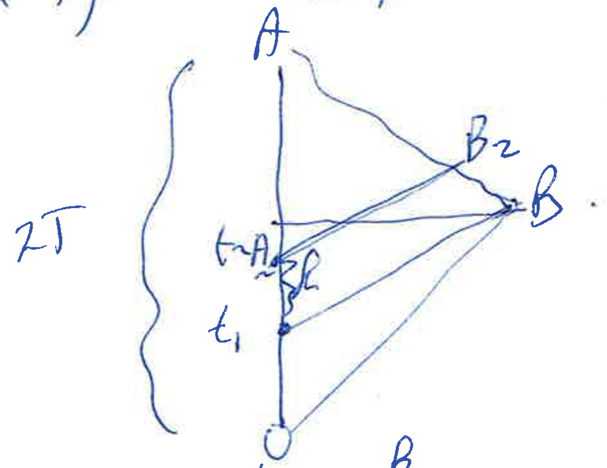
$\tau'_u(t_1) = \sqrt{1-v^2/c^2}\cdot T$

i.e. $\frac{ct_1}{c-V} = \cancel{c}\ T$ or $t_1 = T(1-v/c)$

$t_1 = T(1+v/c)$

and for $\tau'_e(t_1)$ until

For $t_1 > T(1-v/c)$
$\tau_u'(t_1)$ is computed as follows



write $t_2 = t_1 + h$.

Eq of $A_2 B_2$ is $t - t_2 = \frac{1}{c}x$

Intersect AB and Eq. $t - 2T = -\frac{1}{v}x$

at x-value $t_2 + \frac{x}{c} - 2T = -\frac{1}{v}x$.

i.e. $x\left(\frac{1}{c} + \frac{1}{v}\right) = 2T - t_2 = 2T - t_1 - h$

$= 2T - T(1 - v/c) - h$

$= T(1 + v/c) - h$

So t-value for B_2 is

$t_2 + \frac{1}{c}\left[T(1+v/c) - h\right]\frac{cv}{c+v}$

$= T(1-v/c) + h + \frac{v}{c}\left[T(1+v/c) - h\right]\frac{1}{1+v/c}$

$= T - \cancel{Tv/c} + h + \cancel{\frac{v}{c}\cdot T\frac{1-v/c}{1+v/c}} - h\frac{v}{c}\cdot\frac{1}{1+v/c}$.

$= T + h\left(1 - \frac{v}{c}\cdot\frac{1}{1+v/c}\right) + \cancel{T\left\{\frac{v}{c}\cdot\frac{1-v/c}{1+v/c} - v/c\right\}}$

$\cancel{\{\ \} = \beta\frac{1-\beta}{1+\beta} - \beta = \beta\left(\frac{1-\beta}{1+\beta} - 1\right) = \beta\frac{-2\beta}{1+\beta}}$

$\cancel{= -2\beta^2/1+\beta}$

and $= T + h\left(1 - \frac{\beta}{1+\beta}\right)$ $\beta = v/c$

$= T + \frac{h}{1+\beta}$, so when $h = 0$, $t(B_2) = T$

and when $h = 2T - T(1-\beta) = T(1+\beta)$, $T(B_2) = 2T$

as is required.

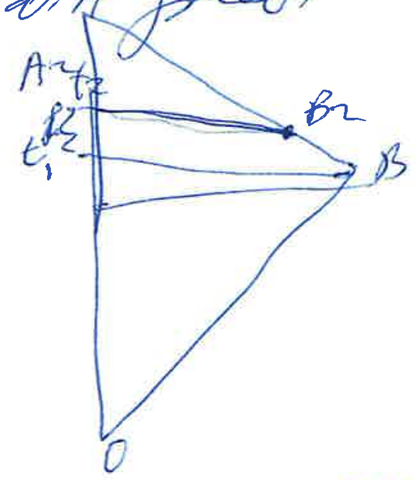
So, in interval of t_1 from $T(1-\beta)$ to $2T$

$$\tau_u(t_1) = \sqrt{1-\beta^2}\cdot T + \sqrt{1-\beta^2}\cdot\frac{h}{1+\beta}$$

slope of this line is $\frac{\sqrt{1-\beta^2}}{1+\beta} = \sqrt{\frac{1-\beta}{1+\beta}}$

While formulae for $\tau_e(t_1)$ applies up to $t_1 = T(1+\beta)$ when it changes as follows

~~Now write~~



Again write $t_2 = t_1 + h$ where now $t_1 = T(1+\beta)$

Eq. of A_2B_2 is $t - t_2 = -\frac{1}{c}x$

Intersects AB with eq. $t - 2T = -\frac{1}{v}x$

at t-value of B_2 (change $c \to -c$ and β)

$$T + \frac{h}{1-\beta}$$

So when $h = 0$, $t(B_2) = T$

and when $h = 2T - T(1+\beta) = T(1-\beta)$, $t(B_2) = 2T$ as we require.

So, in interval of t_1 from $T(1+\beta)$ to $2T$

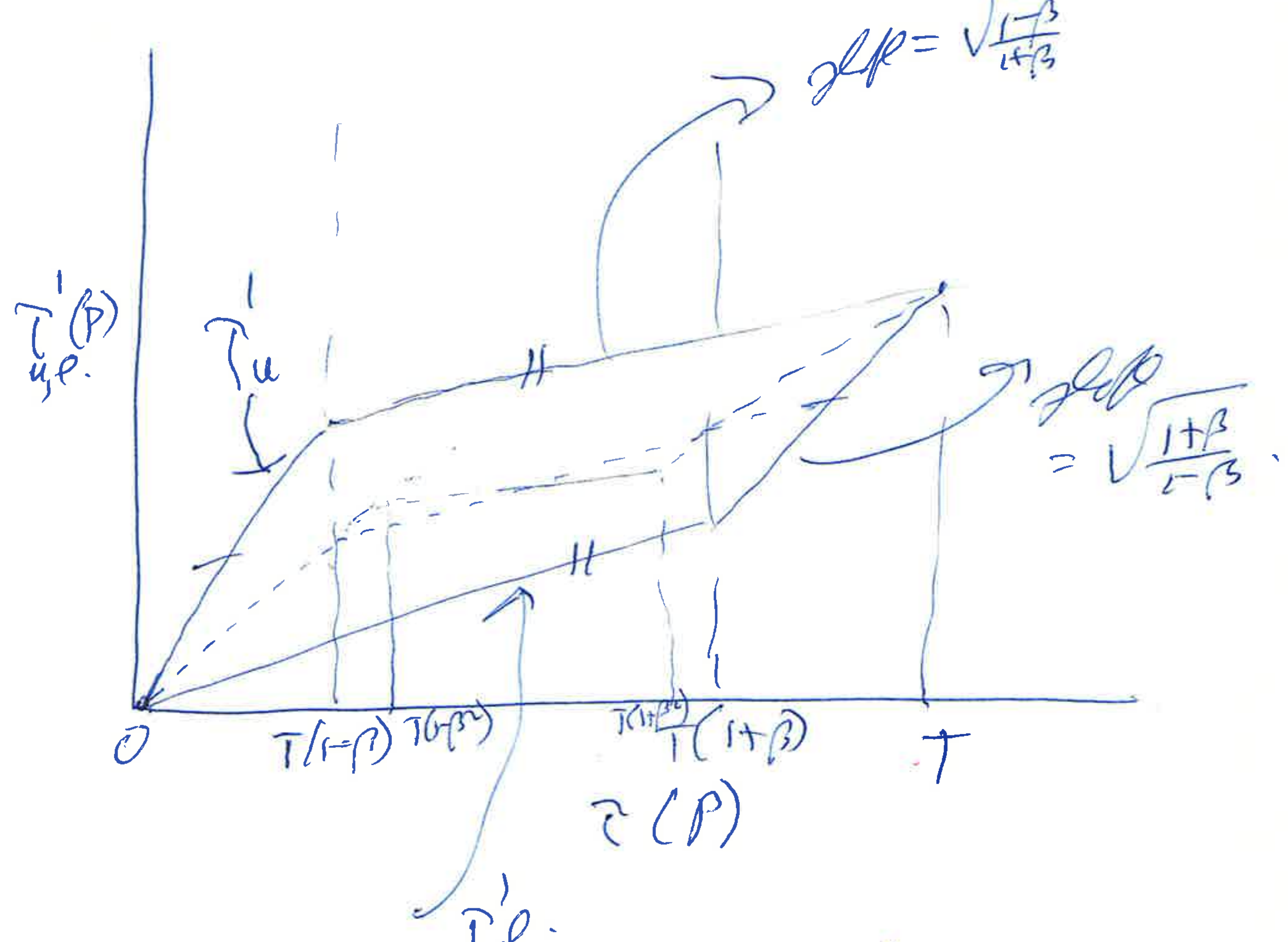
$$\tau_e(t_1) = \sqrt{1-\beta^2}\cdot T + \sqrt{1-\beta^2}\,\frac{h}{1-\beta}.$$

slope of this line is ~~$\frac{\sqrt{1-\beta^2}}{1+\beta} = \sqrt{\frac{1-\beta}{1+\beta}}$~~

$$\frac{\sqrt{1-\beta^2}}{1-\beta} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

So, in summary graph of $\tau'_u(\beta)$ v. $\tau(\beta)$
and $\tau'_\ell(\beta)$ v. $\tau(\beta)$

looks like this:



... but slopes average slope.

From up to $T(1-\beta)$ average slope

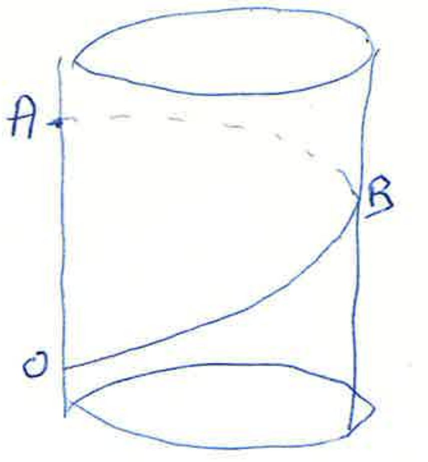
$$= \frac{1}{2}\left[\sqrt{\frac{1-\beta}{1+\beta}} + \sqrt{\frac{1+\beta}{1-\beta}}\right]$$

$$= \frac{1}{2\sqrt{1-\beta^2}}\left[1-\beta + (1+\beta)\right] = \frac{1}{\sqrt{1-\beta^2}}.$$

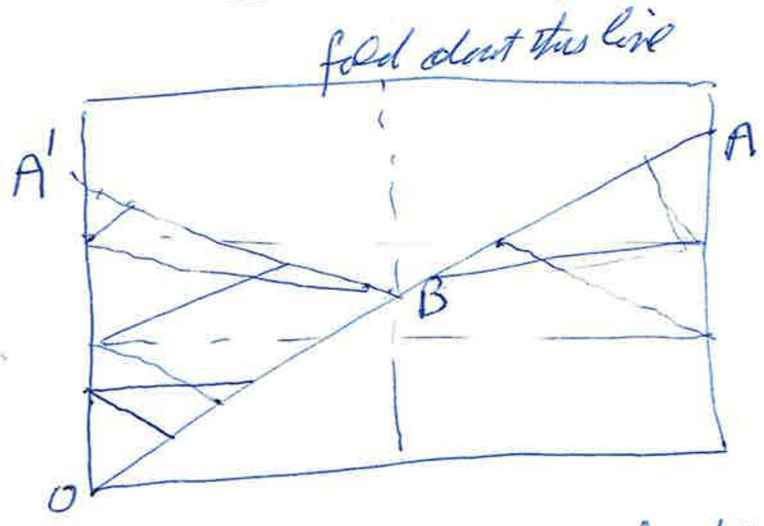
This is also average slope in section $T(1+\beta)$ to $2T$

average slope in stretch $T(1-\beta)$ to $T(1+\beta)$
is just $\sqrt{\frac{1-\beta}{1+\beta}}$ as we found before.

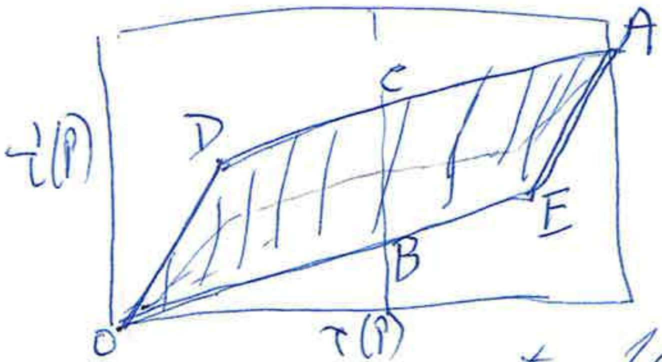
Simultaneity in a cylindrical Universe



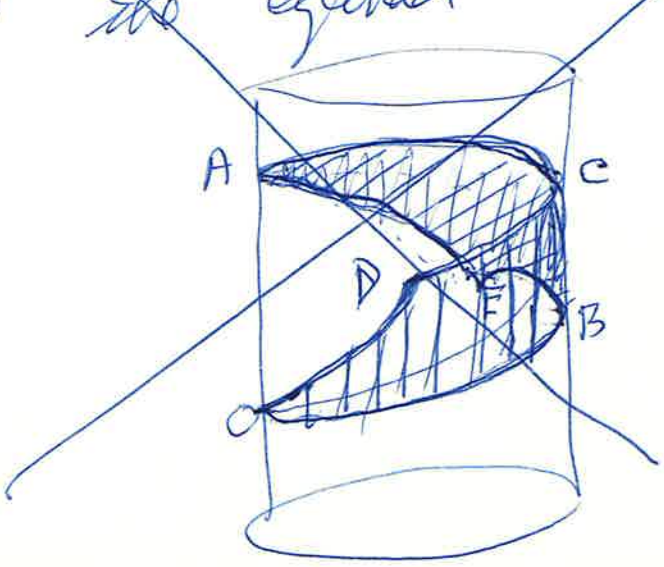
unwrap cylinder

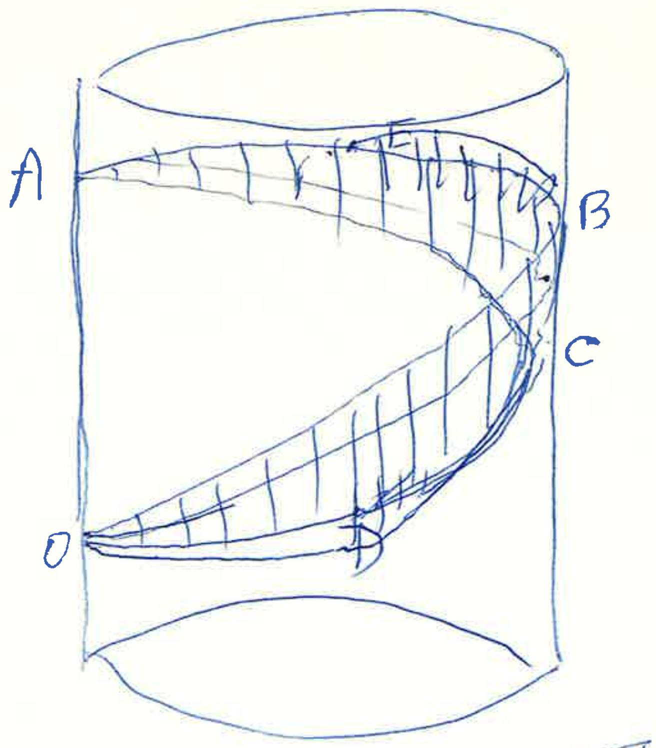


Exactly same diagram for latitude in simultaneity assignments as in the



The parallelogram is now to be wrapped round the cylinder





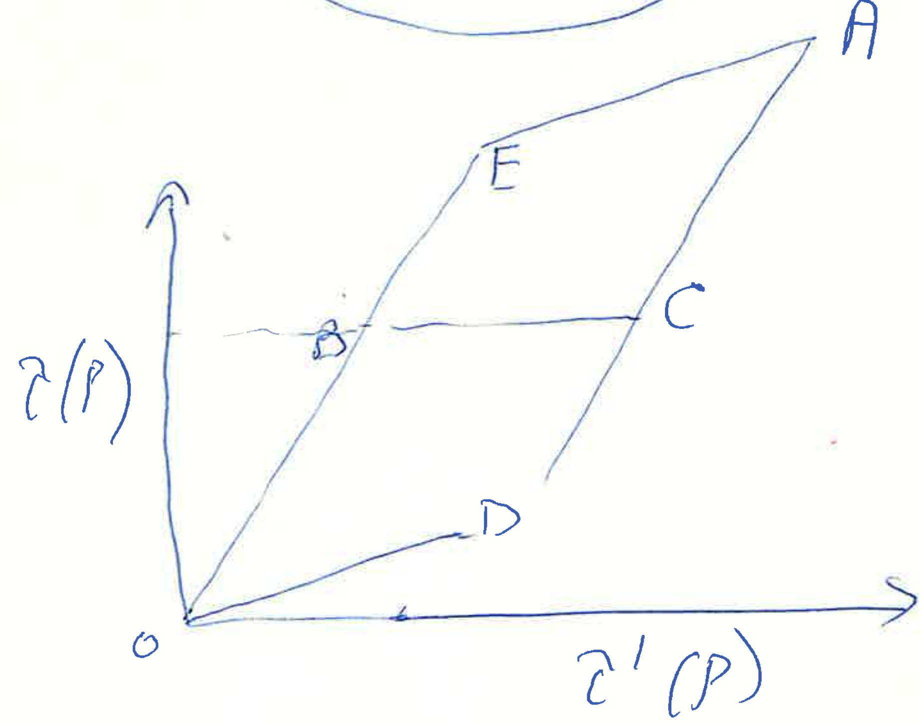
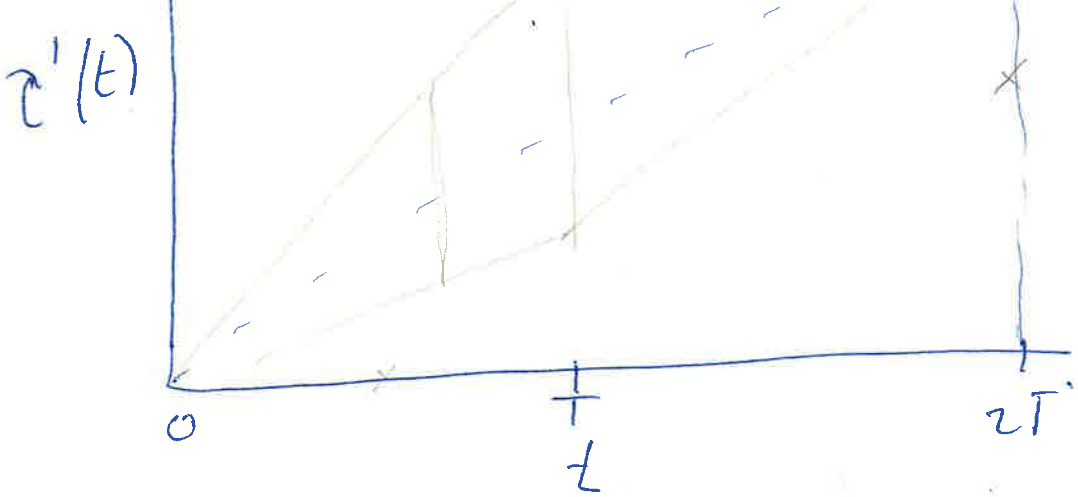
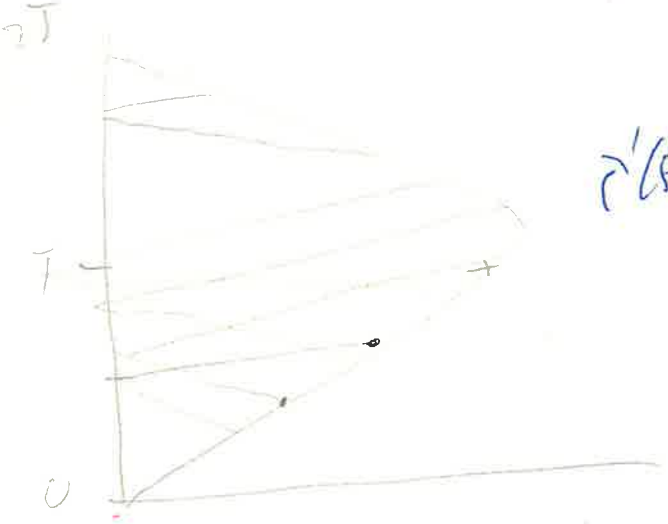
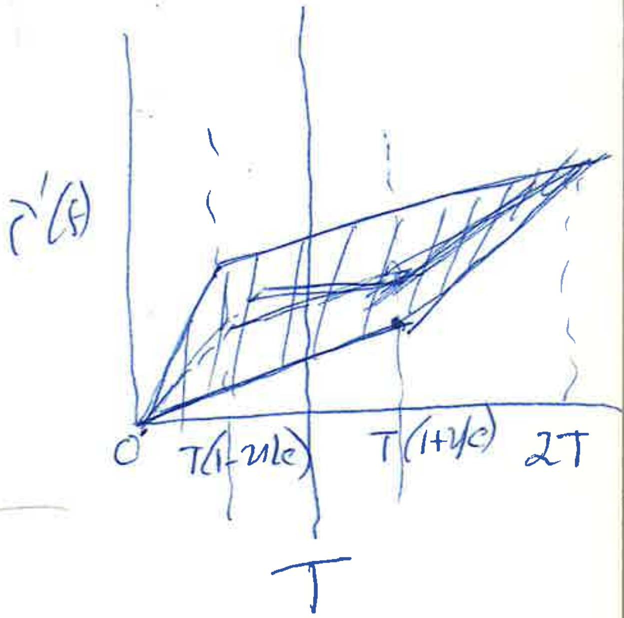
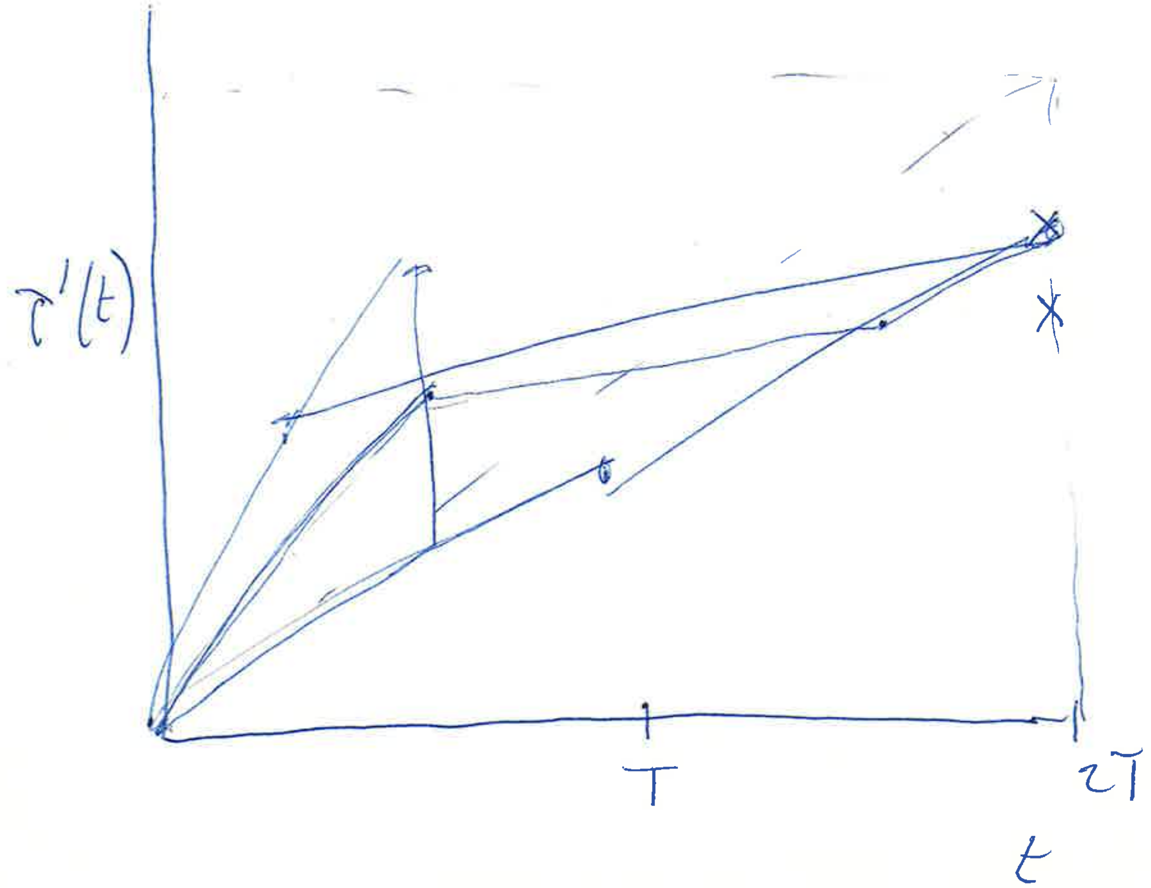


Diagram shows, as we follow path of moving (1), the latitude of $\tau(2)$ that could be allowed for simultaneity of a given point on (1)'s lifeday with time as measured by stationary (2).


$\tau'(t)$
0
T
$2T$
t


$2T$
T
0


$\tau'(t)$
0
$T(1-2u/c)$
$T(1+u/c)$
$2T$
T


$\tau'(t)$
T
$2T$
t

Does Q come before S along τ_β axis

value of τ_β for S is $T(1-v/c)\sqrt{\frac{1+v/c}{1-v/c}} = T\sqrt{1-v^2/c^2}$

value of τ_β for Q is $T(1+v/c)\sqrt{\frac{1-v/c}{1+v/c}} = T\sqrt{1-v^2/c^2}$.

So Fig 8 should look like this

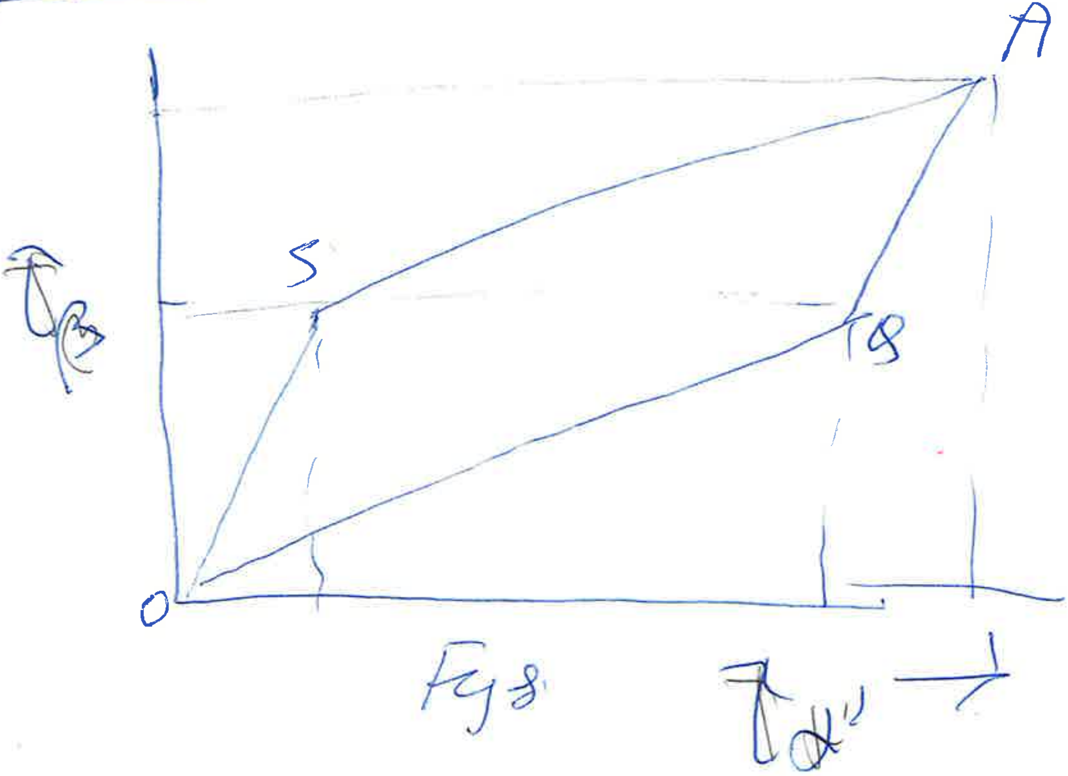


Fig 8

SQ is $\parallel$ τ_α axis

of length $2Tv/c$.

So Fig 9 ordinate ~~coord.~~ of F is $T(1-v^2/c^2)\frac{1}{2}\left(\sqrt{\frac{1+v/c}{1-v/c}} + \sqrt{\frac{1-v/c}{1+v/c}}\right)$

$= T(1-v^2/c^2)\frac{1}{2} \quad \frac{(1+v/c)+(1-v/c)}{\sqrt{1-v^2/c^2}}$